basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P1

NOVEMBER 2022

MARKS: 150

TIME: 3 hours

This question paper consists of 11 pages, a 2-page information sheet and 2 answer sheets.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.

2. Answer ALL the questions.

3. Answer QUESTIONS 4.2.3 and 4.3 on the ANSWER SHEETS provided. Write your centre number and examination number in the spaces provided on the ANSWER SHEETS and hand in the ANSWER SHEETS with your ANSWER BOOK.

4. Number the answers correctly according to the numbering system used in this question paper.

5. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

6. Answers only will NOT necessarily be awarded full marks.

7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

8. If necessary, round off answers to TWO decimal places, unless stated otherwise.

9. Diagrams are NOT necessarily drawn to scale.

10. An information sheet with formulae is included at the end of the question paper.

11. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x(7+x) = 0$ (2)

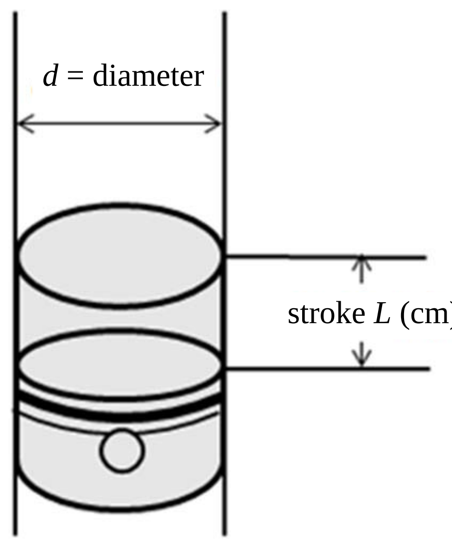
1.1.2 $4x^2 - 5x - 4 = 0$ (correct to TWO decimal places) (3)

1.1.3 $2x^2 - 8 > 0$ (3)

1.2 Solve for x and y if:

$y = 5x - 2$ and $y = x^2 + 4x - 8$ (5)

1.3 The diagram below shows the movement of a piston inside the engine cylinder of a car. Alongside is the formula for calculating the swept volume (SV), which is equal to the base area of the cylinder, multiplied by the length of the stroke (L).



$$SV = \frac{\pi d^2 \times L}{4}$$

Where:

SV = swept volume (in cm^3)
L = length (in cm)
d = diameter (in cm)

1.3.1 Make L the subject of the formula. (2)

1.3.2 Hence, calculate (rounded to the nearest cm), the numerical value of L if $SV = 1\ 020{,}5\ \text{cm}^3$ and the diameter $d = 10$ cm. (2)

1.4 Given the binary numbers:

$$P = 1\ 010_2 \text{ and } Q = 10\ 000_2$$

1.4.1 Write P in decimal form. (1)

1.4.2 Determine $P \times Q$ in binary form. (2)
[20]

QUESTION 2

2.1 Given the equation: $x^2 - 2x + 6 = 0$

2.1.1 Determine the numerical value of the discriminant. (2)

2.1.2 Hence, describe the nature of the roots of the equation. (1)

2.2 Determine the numerical value of k for which the equation $x^2 + 2x + k = 0$ will have real roots. (3)

[6]

QUESTION 3

3.1 Simplify the following **without the use of a calculator**:

3.1.1 $\dfrac{8x^3y^2}{16xy^4}$ (leave the answer with positive exponents) (2)

3.1.2 $\dfrac{\sqrt{48}+\sqrt{12}}{\sqrt{27}}$ (3)

3.2 If $\log 5 = m$, determine the following in terms of m:

3.2.1 $\log 25$ (2)

3.2.2 $\log 2$ (3)

3.3 Solve for x: $\log_2(x+3) - 3 = -\log_2(x-4)$ (5)

3.4 Given complex numbers: $z_1 = -1 + 3i$ and $z_2 = \sqrt{2}\ cis\ 135°$

3.4.1 Write down the conjugate of z_1. (1)

3.4.2 Express z_2 in rectangular form. (2)

3.4.3 Evaluate $z_1 - z_2$. (2)

3.5 Solve for x and y if $x + yi - (1 - i) = 4 + 5i$ (4)

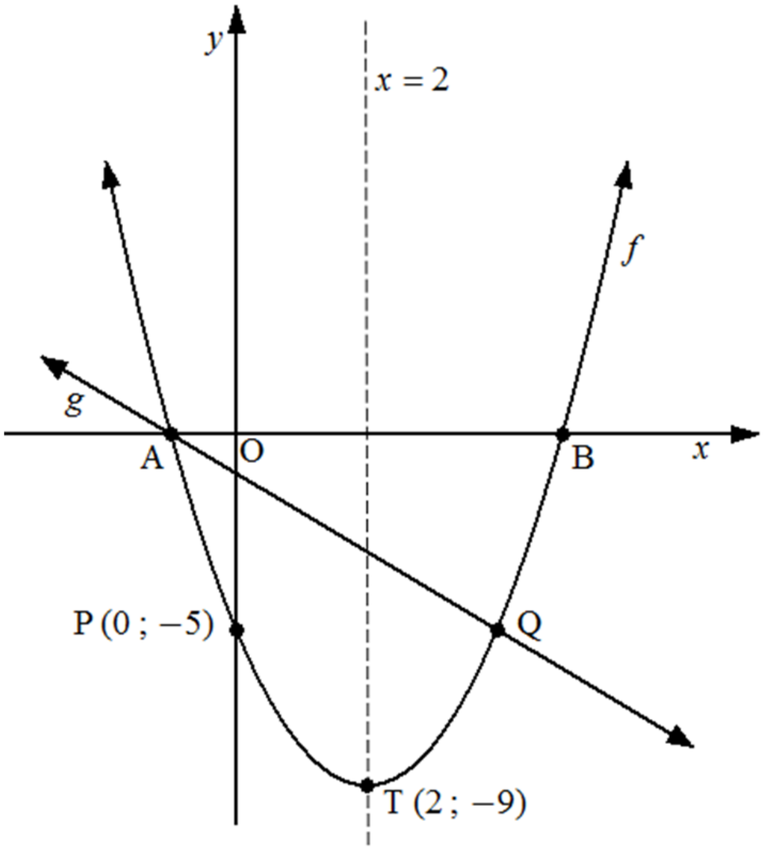
[24]

QUESTION 4

4.1 Sketched below are the graphs of functions f and g defined by:

$f(x) = x^2 - 4x - 5$ and $g(x) = mx + c$

- $T(2\,;-9)$ is the turning point of f.
- Points A, B and $P(0\,;-5)$ are the intercepts of f.
- Q is the reflection of P about the line $x = 2$
- A and Q are the points where $f(x) = g(x)$



4.1.1 Write down:

(a) The range of f (1)

(b) The coordinates of Q (2)

4.1.2 (a) Determine the x-intercept(s) of f. (3)

(b) Hence, write down the length of AB. (1)

4.1.3 Determine the numerical values of m and c. (3)

4.1.4 Write down the value(s) of x for which $f(x) \times g(x) > 0$ (2)

4.2 Given: The functions defined by $h(x) = -\sqrt{13 - x^2}$ and $k(x) = \dfrac{3}{x} + 1$

4.2.1 Write down the domain of function h. (2)

4.2.2 Determine:

(a) The equations of the asymptotes of k (2)

(b) The x-intercept of k (2)

4.2.3 Hence, sketch the graphs of h and k on the same set of axes on the ANSWER SHEET provided. Clearly show the intercepts with the axes and any asymptotes. (5)

4.3 Given: $t(x) = a^x + c$ and the following additional information:

- $y = -1$ is the equation of the asymptote of t
- $a > 1$

Sketch the graph of function t on the set of axes on the ANSWER SHEET provided. Clearly show the intercepts with the axes and the asymptote. (3)

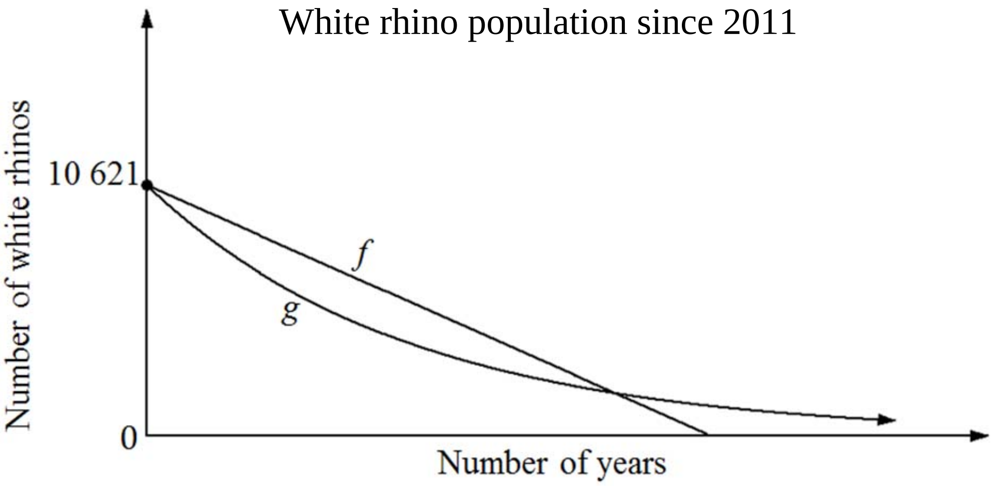
[26]

QUESTION 5

5.1 A cellphone bought in 2022 costed R8 000. Determine the value of a similar cellphone at the end of 3 years if the inflation rate is 13% per annum. (3)

5.2 The white rhino population in the Kruger National Park depreciates at a rate of 12,8% per annum on the reducing-balance method over a period of time (in years).

The information below represents the statistics of a survey done since 2011.



5.2.1 How many white rhinos were there at the start of the survey in 2011? (1)

5.2.2 Which graph (f or g) represents the reducing-balance method ? (1)

5.2.3 Determine (showing ALL calculations) how long it took for the population of white rhinos to decrease to 3 459. Give your answer correct to the nearest year. (5)

5.3 Samuel opened a savings account to save for a boat cruise that he wants to go on at the end of 5 years. He made an initial deposit of R20 000.

- The interest rate for the first 2 years was 6% per annum, compounded monthly.
- At the end of the first 2 years:
  - He deposited a further amount of R5 000
  - The interest rate changed to 5% per annum, compounded half-yearly

Determine (showing ALL calculations) whether he will have enough money in the savings account for the boat cruise will cost R35 000. (5)

[15]

QUESTION 6

6.1 Determine $f'(x)$ using FIRST PRINCIPLES if $f(x) = 5 - 8x$ (5)

6.2 Determine:

6.2.1 $f'(x)$ if $f(x) = 3x^5 + \pi x$ (2)

6.2.2 $\frac{dy}{dx}$ if $y = x^2\left(4x - 2x^{-1}\right)$ (3)

6.2.3 $D_x\left[\sqrt[5]{x^4} - \frac{2}{5x^2} + 8t^4x\right]$ (5)

6.3 The gradient of the tangent to the curve defined by $g(x) = 6x^2 + 3x$ at $x = p$ is -21.

6.3.1 Determine the numerical value of p. (3)

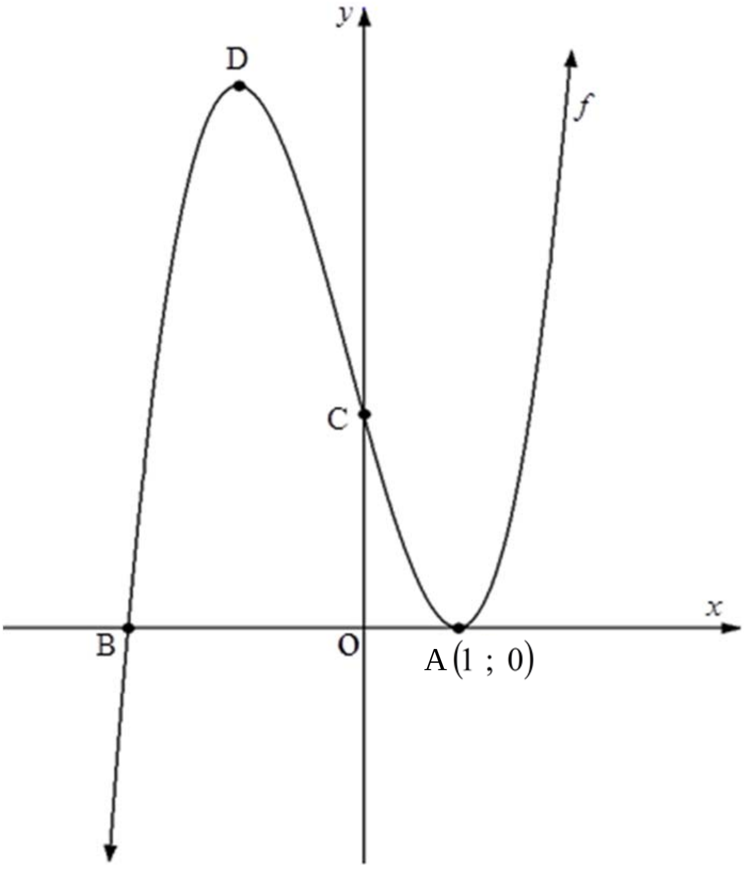
6.3.2 Hence, determine the equation of the tangent to curve g at $x = p$ in the form $y = ...$ (3)

[21]

QUESTION 7

The graph below represents the function defined by $f(x) = x^3 + 3x^2 - 9x + k$ and cuts the x-axis at $A(1\ ;\ 0)$ and B.

The graph cuts the y-axis at C and has turning points at A and D.



7.1 Write down the length of OA. (1)

7.2 Show that $k = 5$ (1)

7.3 Hence, determine the coordinates of point B. (4)

7.4 Determine the coordinates of turning point D. (5)

7.5 Write down the value(s) of x for which $f'(x) \leq 0$ (2)

7.6 If $g(x) = f(x) - 2$, then write down the new coordinates of point A. (2)

[15]

QUESTION 8

An experiment is conducted in which the temperature (T) in degrees Celsius (^{o}C) varies with time (t) in seconds according to the formula:

$$T(t) = 37,5 + 7t - 0,5t^2 \quad \text{where } 0 \leq t \leq 10$$

8.1 Write down the initial temperature. (1)

8.2 Determine the rate of change of the temperature with respect to time when $t = 4$ seconds. (3)

8.3 Determine the maximum temperature reached during the experiment. (3)

8.4 During which time interval was the temperature decreasing? (2)
[9]

QUESTION 9

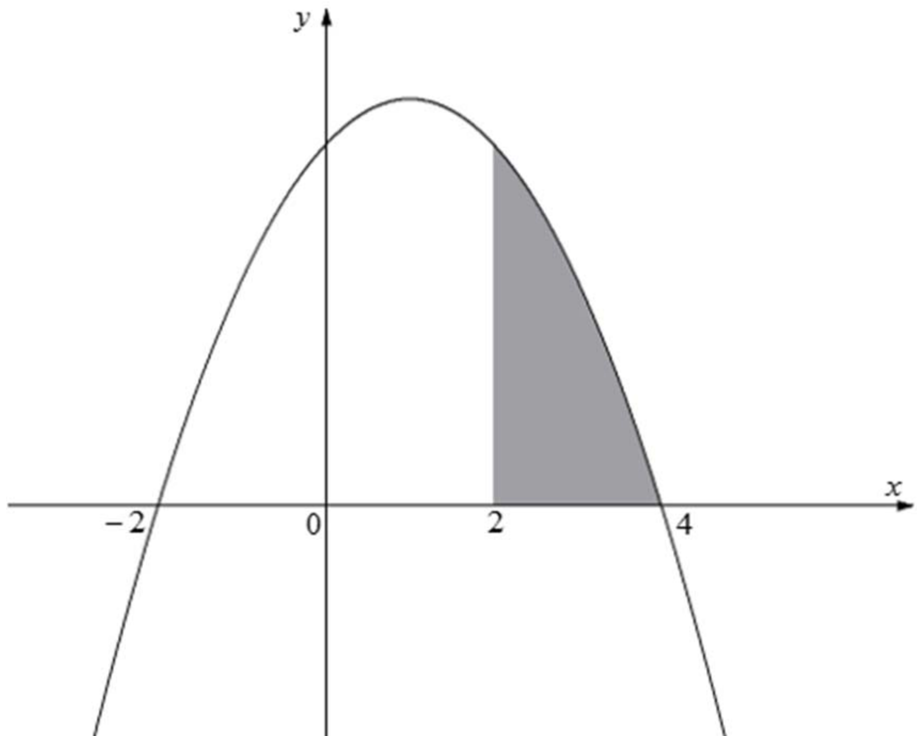
9.1 Determine the following integrals:

9.1.1 $\int 3x^{-1}\ dx$ (2)

9.1.2 $\int \left(4+2^{-x}\right)dx$ (2)

9.1.3 $\int \frac{8x^4 - x^2}{2x}\ dx$ (3)

9.2 The sketch below shows the shaded area bounded by function h defined by $h(x) = -x^2 + 2x + 8$ and the x-axis between the points where $x = 2$ and $x = 4$
The graph of h cuts the x-axis at $x = -2$ and $x = 4$
The area bounded by function h and the x-axis between the x-intercepts is 36 square units.



A learner at a technical high school states that the shaded area is 20% of the area bounded by function h and the x-axis between the x-intercepts.

Is the learner's statement CORRECT? Justify your answer by showing ALL calculations. (7)

[14]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = -\frac{b}{2a} \qquad y = \frac{4ac - b^2}{4a}$$

$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0,\ a \neq 1$ and $b > 0$

$$\mathrm{A} = \mathrm{P}(1 + ni) \qquad \mathrm{A} = \mathrm{P}(1 - ni) \qquad \mathrm{A} = \mathrm{P}(1 + i)^n \qquad \mathrm{A} = \mathrm{P}(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f^{/}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\int k x^n dx = \frac{k x^{n+1}}{n+1} + C \quad , \ n, k \in \mathbb{R}$ with $n \neq -1$ and $k \neq 0$

$\int \frac{k}{x} dx = k \ln x + C \quad , \ x > 0$ and $k \in \mathbb{R}\ ;\ k \neq 0$

$\int k a^{nx} dx = \frac{k a^{nx}}{n \ln a} + C \quad , \ a > 0\ ;\ a \neq 1$ and $k, a \in \mathbb{R}\ ;\ k \neq 0$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_2 + x_1}{2}\ ;\ \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \tan\theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In ΔABC: $\frac{a}{\sin \mathrm{A}} = \frac{b}{\sin \mathrm{B}} = \frac{c}{\sin \mathrm{C}}$ $\qquad a^2 = b^2 + c^2 - 2bc \cdot \cos \mathrm{A}$

area of Δ ABC $= \frac{1}{2} ab \cdot \sin \mathrm{C}$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \mathrm{cosec}^2\,\theta$$

$\pi \, rad = 180°$

Angular velocity $= \omega = 2\pi n$ where n = rotation frequency

Angular velocity $= \omega = 360° \, n$ where n = rotation frequency

Circumferential velocity $= v = \pi D n$ where D = diameter and n = rotation frequency

Circumferential velocity $= v = \omega r$ where ω = angular velocity and r = radius

Arc length $= s = r\theta$ where r = radius and θ = central angle in radians

Area of a sector $= \dfrac{r s}{2}$ where r = radius, s = arc length

Area of a sector $= \dfrac{r^2 \theta}{2}$ where r = radius and θ = central angle in radians

$4h^2 - 4dh + x^2 = 0$ where h = height of segment, d = diameter of circle and x = length of chord

$A_T = a(m_1 + m_2 + m_3 + ... + m_n)$ where a = number of equal parts, $m_1 = \dfrac{o_1 + o_2}{2}$

$O_n = n^{th}$ ordinate and n = number of ordinates

OR

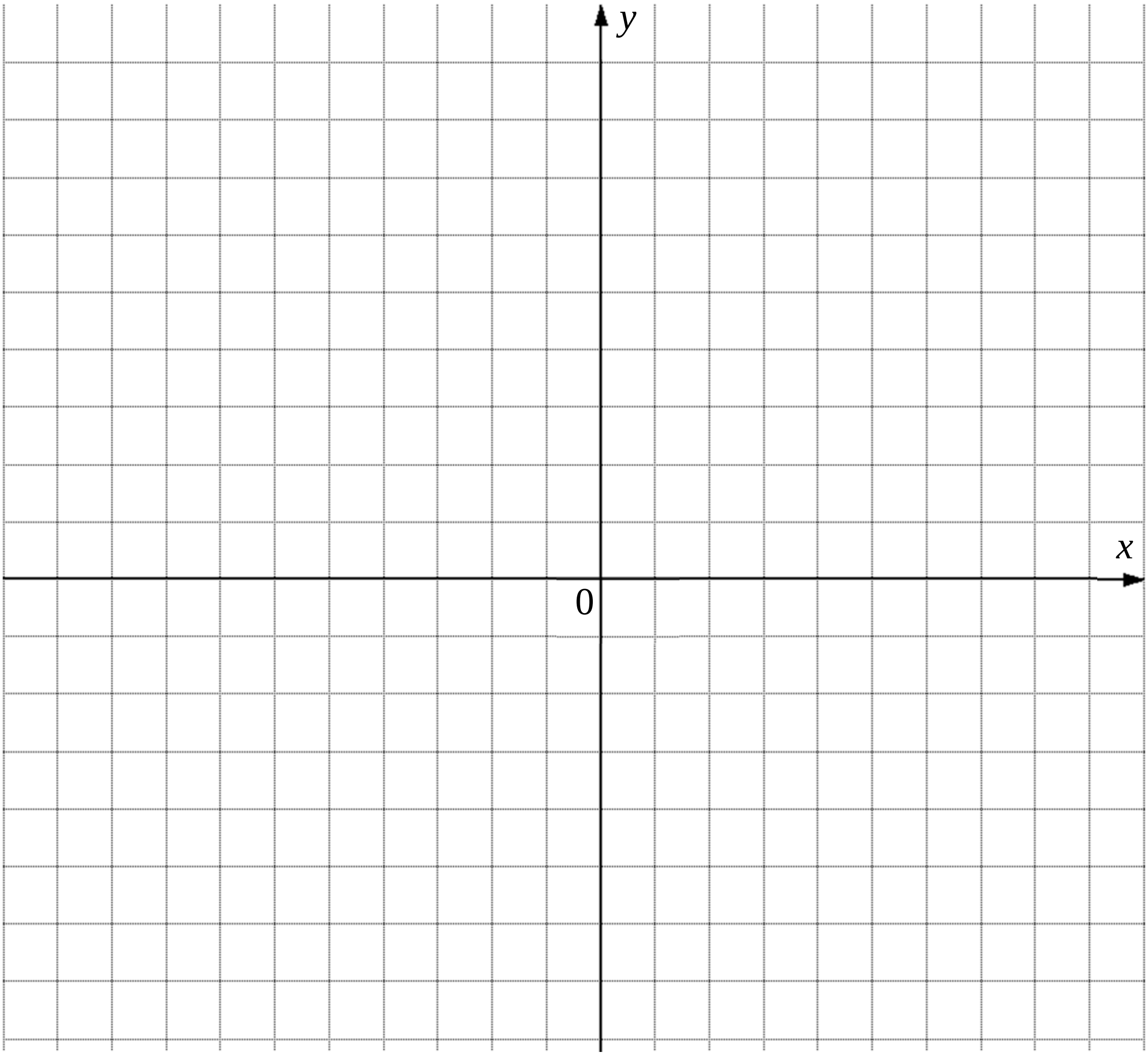
$A_T = a\left(\dfrac{o_1 + o_n}{2} + o_2 + o_3 + ... + o_{n-1}\right)$ where a = number of equal parts, $o_n = n^{th}$ ordinate and n = number of ordinates

ANSWER SHEET

CENTRE NUMBER							

EXAMINATION NUMBER													

QUESTION 4.2.3



ANSWER SHEET

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QUESTION 4.3